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DOUBLE-DIFFUSIVE LAMINAR NATURAL CONVECTION IN TWO-DIMENSIONAL POROUS CAVITIES

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Abstract. *This paper presents an analysis of the macroscopic heat and mass transport equations for laminar flow in permeable structures. Two driving mechanisms are considered to contribute to the overall momentum transport, namely temperature driven and concentration driven mass fluxes. Double-diffusive natural convection mechanism is investigated for the fluid phase in laminar regime. Equations are presented based on the double-decomposition concept, which considers both time fluctuations and spatial deviations about mean values. This work intends to demonstrate that additional transport mechanisms are mathematically derived if temperature, concentration and velocity present simultaneously time fluctuations and spatial deviations within the domain of analysis. Stability analysis of mixtures composed of lighter or heavier components under gradients of temperature and concentration is discussed upon.*

Keywords*. Natural convection, Double-diffusive, Laminar flow, Double-decomposition.*

1. Introduction

The study of double-diffusive natural convection in porous media has many environmental and industrial applications, including grain storage and drying, petrochemical processes, oil and gas extraction, contaminant dispersion in underground water reservoirs, electrochemical processes, etc (Bennacer et al, 2001, Goyeau et al, 1996, Mamou et al, 1995, Mamou et al, 1998) and (Mohamad and Bennacer, 2002). In some specific applications, the fluid mixture may become turbulent and difficulties arise in the proper mathematical modeling of the transport processes under both temperature and concentration gradients. Modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat (Hsu and Cheng, 1990) or mass transfer (Bear, 1972, Bear and Bachmat, 1967, Whitaker, 1966 and Whitaker, 1967).

If time fluctuations of the flow properties are considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: a) application of time-average operator followed by volume-averaging (Kuwahara et al, 1996, Kuwahara and Nakayama, 1998), or b) use of volume-averaging before time-averaging is applied, (Lee and Howell, 1987). This work intends to present a set of macroscopic mass transport equations derived under the recently established double-decomposition concept (Pedras and de Lemos, 2000, Pedras and de Lemos, 2001, Pedras and de Lemos, 2001b, Pedras and de Lemos, 2001c), trough which the connection between the two paths a) and b) above is unveiled. That methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature (Rocamora and de Lemos, 2000). Buoyant flows (de Lemos and Braga, 2003) and mass transfer (de Lemos and Mesquita, 2003) have also been investigated. Recently, a general classification of all proposed models for turbulent flow and heat transfer in porous media has been published (de Lemos and Pedras, 2001).

Motivated by the foregoing, this paper intends to validate the present numerical tool using in this preliminarily only the laminar equations. Future improvements intend to analyses the turbulent behavior of the double diffusive phenomena since the laminar regime has shown good agreement with those results obtained by other authors in the open literature.

2. Local Instataneous Transport Equation

The steady-state microscopic instantaneous transport equations for an incompressible binary fluid mixture with constant properties are given by:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho \nabla \cdot (\mathbf{uu}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \tag{2}$$

$$(\rho c_p) \nabla \cdot (\mathbf{u}T) = \nabla \cdot (\lambda \nabla T) \tag{3}$$

$$\rho\nabla\cdot(\mathbf{u}\,m_\ell+\mathbf{J}_\ell)=\rho\,R_\ell \tag{4}$$

where $\mathbf{u}$ is the mass-averaged velocity of the mixture, $\mathbf{u}=\sum_\ell m_\ell\mathbf{u}_\ell$, $\mathbf{u}_\ell$ is the velocity of species ℓ, m_ℓ is the mass fraction of component ℓ, defined as $m_\ell=\rho_\ell/\rho$, ρ_ℓ is the mass density of species ℓ (mass of ℓ over total mixture volume), ρ is the bulk density of the mixture ($\rho=\sum_\ell\rho_\ell$), p is the pressure, μ is the fluid mixture viscosity, $\mathbf{g}$ is the gravity acceleration vector, c_p is the specific heat, T is the temperature and λ is the fluid thermal conductivity. The generation rate of species ℓ per unit of mixture mass is given in (4) by R_ℓ.

An alternative way of writing the mass transport equation is using the volumetric molar concentration C_ℓ (mol of ℓ over total mixture volume), the molar weight M_ℓ (g/mol of ℓ) and the molar generation/destruction rate R_ℓ^* (mol of ℓ /total mixture volume), giving:

$$M_\ell\,\nabla\cdot(\mathbf{u}\,C_\ell+\mathbf{J}_\ell)=M_\ell\,R_\ell^* \tag{5}$$

Further, the mass diffusion flux J_ℓ (mass of ℓ per unit area per unit time) in (4) or (5) is due to the velocity slip of species ℓ,

$$\mathbf{J}=\rho_\ell(\mathbf{u}_\ell-\mathbf{u})=-\rho_\ell D_\ell\nabla m_\ell=-M_\ell D_\ell\nabla C_\ell \tag{6}$$

where D_ℓ is the diffusion coefficient of species ℓ into the mixture. The second equality in Eq. (6) is known as Fick's Law, which is a constitutive equation strictly valid for binary mixtures under the absence of any additional driving mechanisms for mass transfer (Hsu and Cheng, 1990). Therefore, no Soret or Dufour effects are here considered.

Rearranging (5) for an inert species, dividing it by M_ℓ and dropping the index ℓ for a simple binary mixture, one has,

$$\nabla\cdot(\mathbf{u}\,C)=\nabla\cdot(D\,\nabla C) \tag{7}$$

If one considers that the density in the last term of (2) varies with temperature and concentration, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density ρT,

$$\rho_T\cong\rho[1-\beta(T-T_{ref})-\beta_C(C-C_{ref})] \tag{8}$$

where the subscript ref indicates a reference value and β and β_C are the thermal and salute expansion coefficients, respectively, defined by,

$$\beta=-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\bigg|_{p,C},\quad \beta_C=-\frac{1}{\rho}\frac{\partial\rho}{\partial C}\bigg|_{p,T} \tag{9}$$

Equation (8) is an approximation of (9) and shows how density varies with temperature and concentration in the body force term of the momentum equation.

Further, substituting (8) into (2), one has,

$$\rho\nabla\cdot(\mathbf{uu})=-\nabla p+\mu\nabla^2\mathbf{u}+\rho\,\mathbf{g}[1-\beta(T-T_{ref})-\beta(C-C_{ref})] \tag{10}$$

Thus, the momentum equation becomes,

$$\rho\nabla\cdot(\mathbf{uu})=-(\nabla p)^*+\mu\nabla^2\mathbf{u}-\rho\,\mathbf{g}[(\beta(T-T_{ref})+\beta_C(C-C_{ref})] \tag{11}$$

where $(\nabla p)^*=\nabla p-\rho\,\mathbf{g}$ is a modified pressure gradient.

As mentioned, there are, in principle, two ways that one can follow in order to treat turbulent flow in porous media. The first method applies a time average operator to the governing Eq. (4) before the volume average procedure is

conducted. In the second approach, the order of application of the two average operators is reversed. Both techniques aim at derivation of a suitable macroscopic turbulent mass transport equation.

Volume averaging in a porous medium, described in detail in references (Gray and Lee, 1997) and (Whitaker, 1969), makes use of the concept of a Representative Elementary Volume (REV), over which local equations are integrated. After integration, detailed information within the volume is lost and, instead, overall properties referring to a REV are considered. In a similar manner, statistical analysis of turbulent flow leads to time mean properties. Transport equations for statistical values are considered in lieu of instantaneous information on the flow.

Before undertaking the task of developing macroscopic equations, it is convenient to recall the definition of time average and volume average.

2.1. Volume Average Operator

The volume average of φ taken over a Representative Elementary Volume in a porous medium can be written as:

$$\langle\varphi\rangle^{v}=\frac{1}{\Delta V}\int_{\Delta V}\varphi\, dV \tag{12}$$

The value $\langle\varphi\rangle^{v}$ is defined for any point x surrounded by a Representative Elementary Volume, of size ΔV. This average is related to the intrinsic average for the fluid phase as:

$$\langle\varphi_f\rangle^{v}=\phi\langle\varphi_f\rangle^{i} \tag{13}$$

where $\phi=\Delta V_f/\Delta V$ is the medium porosity and ΔV_f is the volume occupied by the fluid in a REV. Furthermore, one can write:

$$\varphi=\langle\varphi\rangle^{i}+{}^{i}\varphi \tag{14}$$

with $\langle{}^{i}\varphi\rangle^{i}=0$. In Eq. (14), ${}^{i}\varphi$ is the spatial deviation of φ with respect to the intrinsic average $\langle\varphi\rangle^{i}$.

Further, the local volume average theorem can be expressed as (Gray and Lee, 1997) and (Whitaker, 1969):

$$\begin{aligned}
\langle\nabla\varphi\rangle^{v} &= \nabla(\phi\langle\varphi\rangle^{i})+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\varphi dS\\
\langle\nabla\cdot\boldsymbol{\varphi}\rangle^{v} &= \nabla\cdot(\phi\langle\boldsymbol{\varphi}\rangle^{i})+\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot\boldsymbol{\varphi}\, dS\\
\langle\frac{\partial\varphi}{\partial t}\rangle^{v} &= \frac{\partial}{\partial t}(\phi\langle\varphi\rangle^{i})-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\mathbf{u}_i\varphi)dS
\end{aligned} \tag{15}$$

where $\mathbf{n}$ is the unit vector normal to the fluid-solid interface and A_i is the fluid-solid interface area within the REV. It is important to emphasize that Ai should not be confused with the surface area surrounding volume ΔV.

2.2. Macroscopic Equations for Buoyancy Free Flows

Momentum transport

$$\rho\nabla\cdot\left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)=-\nabla(\phi\langle p\rangle^{i})+\mu\nabla^2\mathbf{u}_D-\left[\frac{\mu\phi}{K}\mathbf{u}_D+\frac{c_F\phi\rho|\mathbf{u}_D|\,\mathbf{u}_D}{\sqrt{K}}\right] \tag{16}$$

Heat transport

$$(\rho c_p)_f\,\nabla\cdot(\mathbf{u}_D\langle T\rangle^{i})=\nabla\cdot\{\mathbf{K}_{eff}\cdot\nabla\langle T\rangle^{i}\} \tag{17}$$

$$\mathbf{K}_{eff}=[\phi\lambda_f+(1-\phi)\lambda_s]\mathbf{I}+\mathbf{K}_{tor}+\mathbf{K}_{disp} \tag{18}$$

The subscripts f and s refer to fluid and solid phases, respectively, and coefficients K's come from the modeling of the following mechanisms:

- Tortuosity: $\left[\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\left(\lambda_f T_f - \lambda_s T_s\right)dS\right] = \mathbf{K}_{tor}\cdot\nabla\langle T\rangle^i$ (19)

- Thermal dispersion: $-(\rho c_p)_f\ \phi\langle {}^i\mathbf{u}\,{}^iT_f\rangle^i = \mathbf{K}_{disp}\cdot\nabla\langle T\rangle^i$ (20)

Mass transport

$$\nabla\cdot(\mathbf{u}_D\langle C\rangle^i) = \nabla\cdot\mathbf{D}_{eff}\cdot\nabla(\phi\langle C\rangle^i) \tag{21}$$

$$\mathbf{D}_{eff} = \mathbf{D}_{disp} + \mathbf{D}_{diff} \tag{22}$$

$$\mathbf{D}_{diff} = \langle D\rangle^i\mathbf{I} = \frac{1}{\rho}\frac{\mu_\phi}{Sc}\mathbf{I} \tag{23}$$

The coefficients $\mathbf{D}_{disp}$ in (21) appear due to the nonlinearity of the convection term. They come from the modeling of the following mechanisms:

- Mass dispersion: $-\langle {}^i\mathbf{u}\ {}^iC\rangle^i = \mathbf{D}_{disp}\cdot\nabla\langle C\rangle^i$ (24)

where here we do not account for the thermal and mass dispersion effects, taking in to account only the diffusive and convective ones. Thus, $\mathbf{K}_{tor}$, $\mathbf{K}_{disp}$ and $\mathbf{D}_{disp}$ are null.

2.3. Macroscopic Double-Diffusion Effects

Mean Flow

Focusing now attention to buoyancy effects only, application of the volume average procedure to the last term of (11) leads to,

$$\langle\rho\mathbf{g}[\beta(T-T_{ref})+\beta_C(C-C_{ref})]\rangle^v = \frac{\Delta V_f}{\Delta V}\frac{1}{\Delta V_f}\int_{\Delta V_f}\rho\mathbf{g}[\beta(T-T_{ref})+\beta_C(C-C_{ref})]\,dV \tag{25}$$

Expanding the left hand side of (25) in light of (14), the buoyancy term becomes,

$$\begin{aligned}&\langle\rho\mathbf{g}[\beta(T-T_{ref})+\beta_C(C-C_{ref})]\rangle^v\\ &\quad=\rho\mathbf{g}\phi[\beta_\phi(\langle T\rangle^i-T_{ref})+\beta_{C_\phi}(\langle C\rangle^i-C_{ref})]+\underbrace{\rho\mathbf{g}\beta\phi\left\langle {}^iT\right\rangle^i}_{=0}+\underbrace{\rho\mathbf{g}\beta_C\phi\left\langle {}^iC\right\rangle^i}_{=0}\end{aligned} \tag{26}$$

where the third and forth terms on the r.h.s. are null since $\langle {}^i\varphi\rangle^i = 0$. Here, coefficients β_ϕ and β_{C_ϕ} are the macroscopic thermal and salute expansion coefficients, respectively. Assuming that gravity is constant over the REV, expressions for them based on (26) are given as,

$$\beta_\phi = \frac{\langle\rho\beta(T-T_{ref})\rangle^v}{\rho\phi(\langle T\rangle^i-T_{ref})};\ \beta_{C_\phi} = \frac{\langle\rho\beta_C(C-C_{ref})\rangle^v}{\rho\phi(\langle C\rangle^i-C_{ref})} \tag{27}$$

Including (26) into (16), the macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as,

$$\rho\nabla\cdot\left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)=-\nabla(\phi\langle p\rangle^i)+\mu\nabla^2\mathbf{u}_D+\rho\mathbf{g}\phi[\beta_\phi(\langle T\rangle^i-T_{ref})+\beta_{C_\phi}(\langle C\rangle^i-C_{ref})]-\left[\frac{\mu\phi}{K}\mathbf{u}_D+\frac{c_F\phi\rho|\mathbf{u}_D|\,\mathbf{u}_D}{\sqrt{K}}\right] \quad (28)$$

3. Results and Conclusions

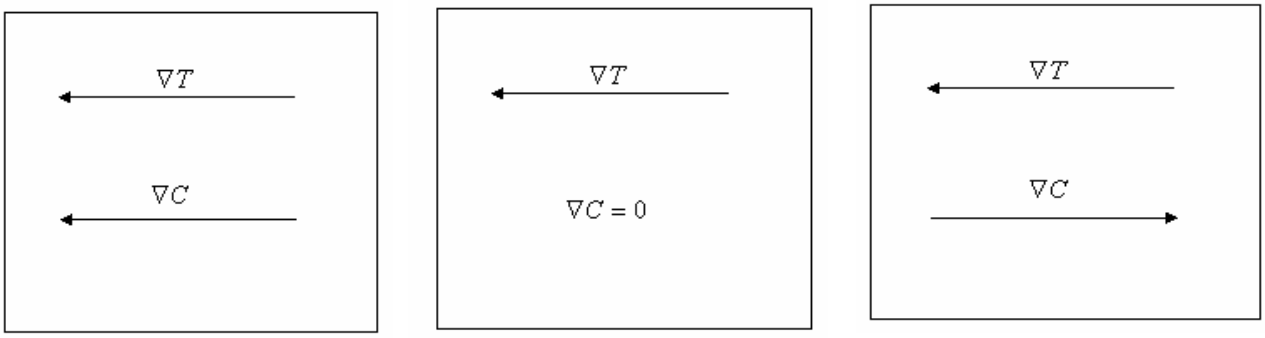
The presents work refers to the study of natural convective flows in a porous cavity (height H, width L: aspect ratio $A=H/L$), saturated by a binary fluid (such as aqueous solutions, as in numerous experimental studies related to solidification processes). Horizontal temperature and concentration differences are specified between the vertical walls (T_1 and C_1 on the left wall, T_2 and C_2 on the right surface), and zero mass and heat fluxes are imposed at the horizontal wall. Figure 1 shows different combinations of temperature and mass concentration gradients. All boundaries are considered to be impermeable. The binary fluid is assumed to be Newtonian and to satisfy the Boussinesq approximation; the flow is incompressible, laminar, 2D and in the steady state.

In this work, equations were derived for laminar double-diffusive natural convection in porous media. Derivations were carried out under the light of the double decomposition concept [0-0]. Extra terms appearing in the equations needed to be modeled in terms of $\mathbf{u}_D$, $\langle T\rangle$ and $\langle C\rangle$.

Figures 2 to 4 shows temperature and mass concentration fields for different values of N, with gradients in different relative positions, using $Ra^*=100$, $Le=100$ and $Da=10^{-3}$. We can observe that with an increase in N the structure of the flow changes significantly. For low values of N, the entire cavity is affected by the driven flow and the boundary layer regime gradually appears with the increase of N. These changes on the flow pattern have a direct consequence on the concentration fields.

Table 1 presents the values of calculated average Nusselt and Sherwood numbers compared with those obtained by Trevisan and Bejan [0] and B. Goyeau [0]. As it can be seen, good agreement was obtained when the present values were compared with literature results. Tables 2, 3 e 4 present values of average Nusselt and Sherwood, for *N*=0.1, 1 and 10, respectively, with *Le*=10 and *A*=1. We can observe that changes on *N* do not affect much integral parameters of the flow, at lease for the conditions here simulated.

On the overall, we can conclude that results herein agree qualitatively well with published data in the literature.



a) b) c)

Figure 1– Problem Geometry: a) $\nabla T=-1,\nabla C=-1$; b) $\nabla T=-1,\nabla C=0$; c) $\nabla T=-1,\nabla C=+1$.

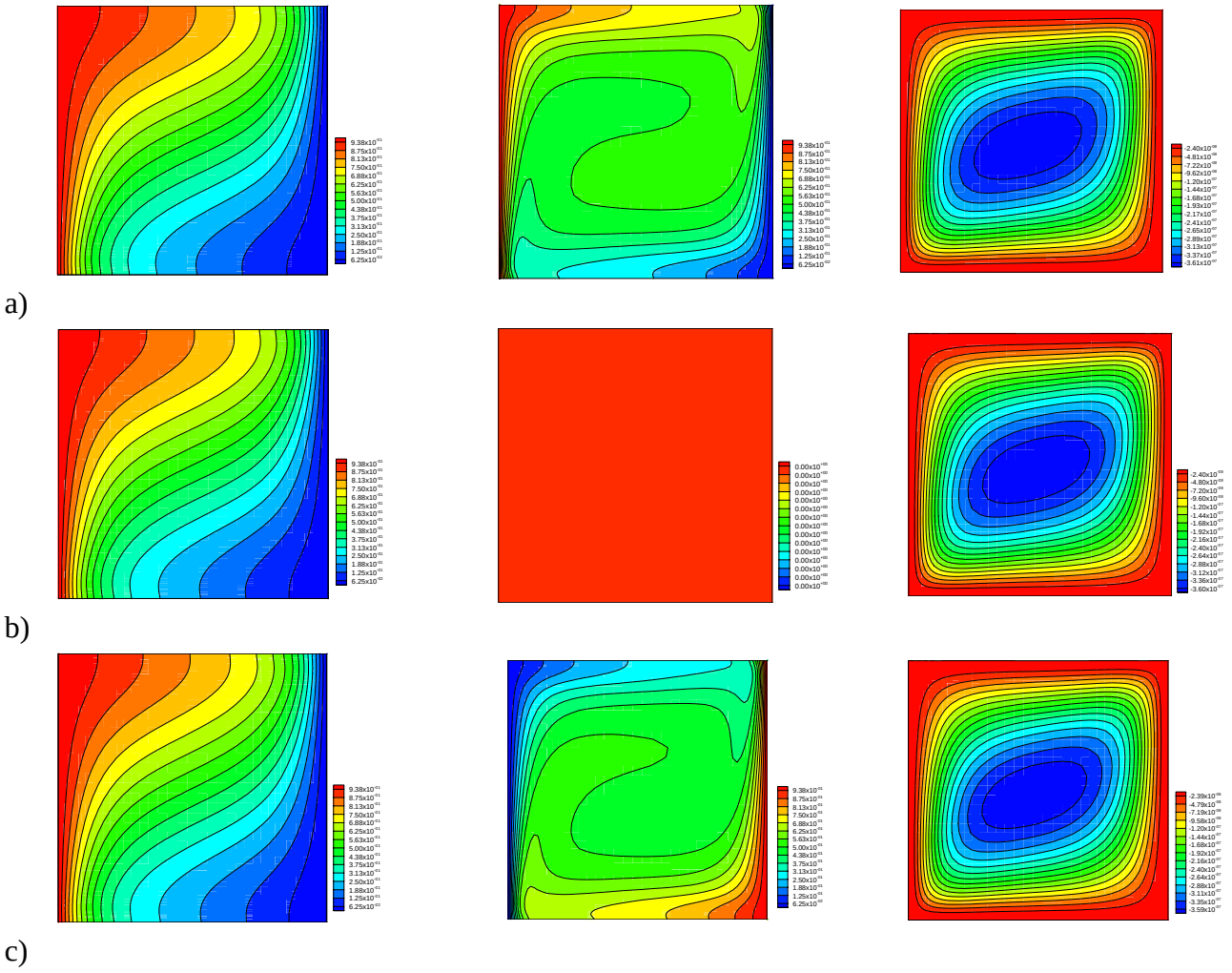


Figure 2: Isotherms, isoconcentration and streamlines lines ($N = 0.1$; $Ra^* = 100$; $Le = 100$; $A = 1$, $Da = 10^{-3}$): a) $\nabla T = -1, \nabla C = -1$; b) $\nabla T = -1, \nabla C = 0$; c) $\nabla T = -1, \nabla C = +1$.

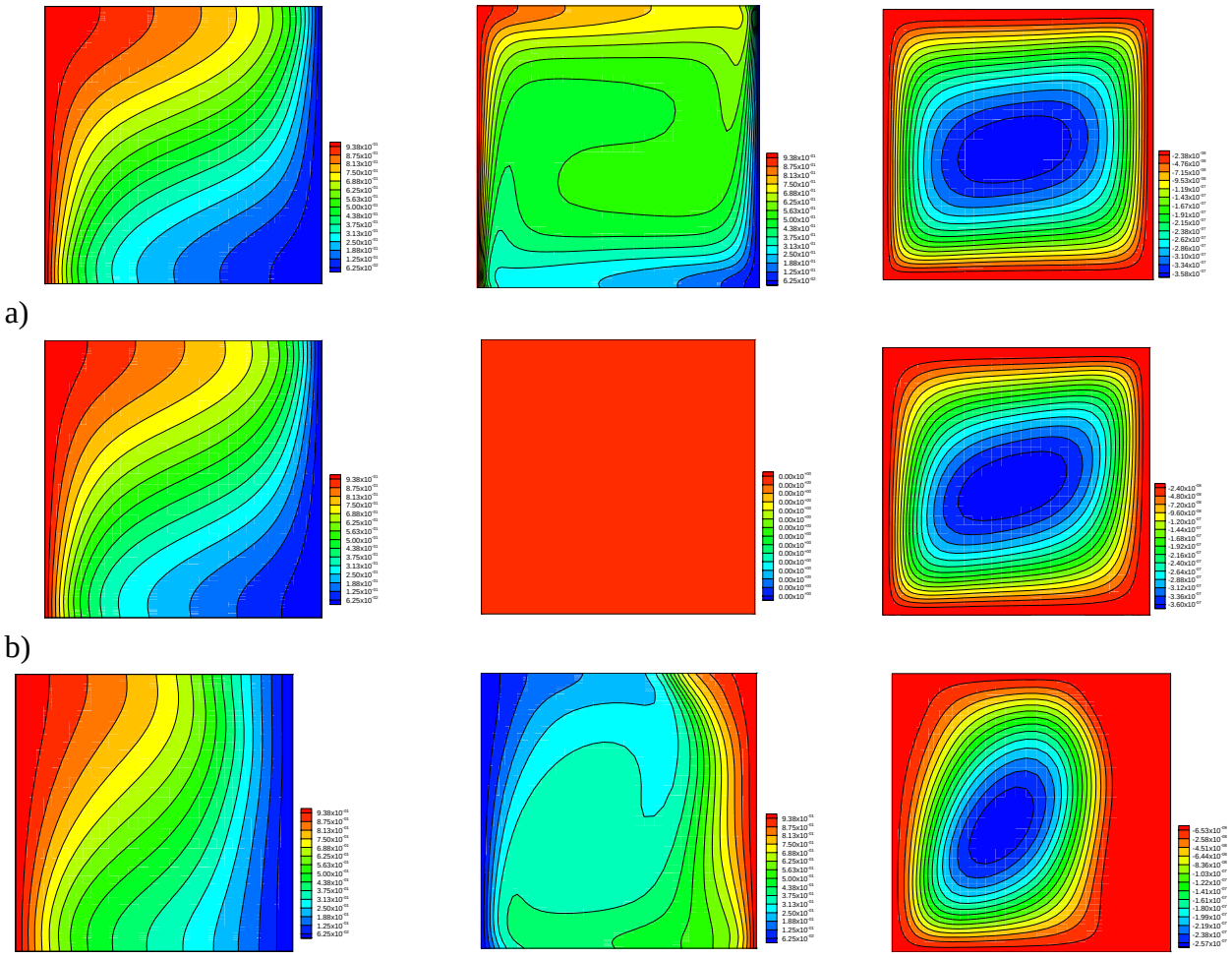


Figure 3– Isotherms, isoconcentration and streamlines lines ($N = 1$; $Ra^* = 100$; $Le = 100$; $A = 1$, $Da = 10^{-3}$): a) $\nabla T = -1, \nabla C = -1$; b) $\nabla T = -1, \nabla C = 0$; c) $\nabla T = -1, \nabla C = +1$.

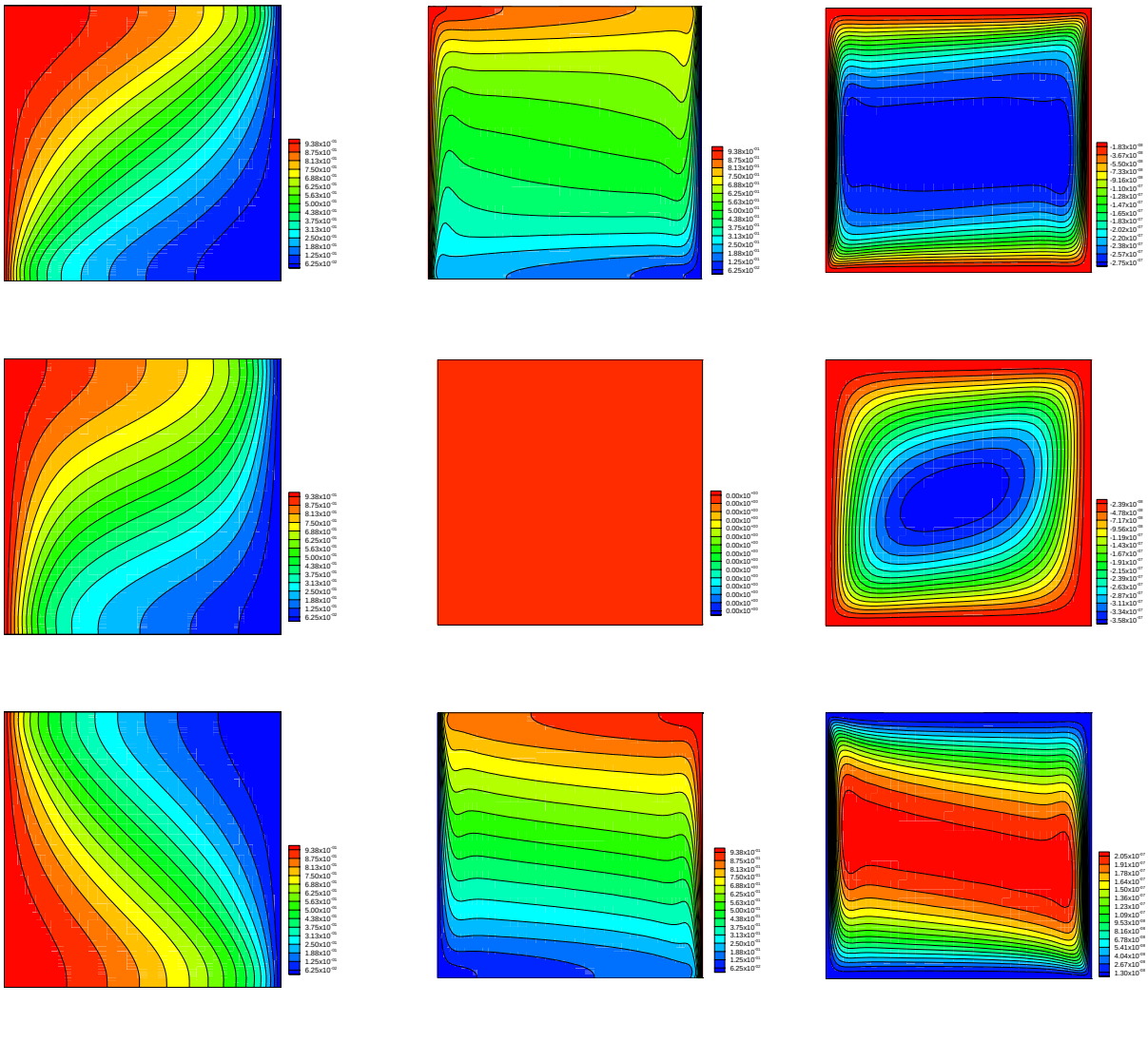


Figure 4– Isotherms, isoconcentration and streamlines lines ($N = 10$; $Ra^* = 100$; $Le = 100$; $A = 1$, $Da = 10^{-3}$): a) $\nabla T = -1, \nabla C = -1$; b) $\nabla T = -1, \nabla C = 0$; c) $\nabla T = -1, \nabla C = +1$.

Table 1: Average Nusselt and Sherwood numbers (N=0 only thermal drive, Le=10, A=1).

Ra^*	Gradients Types		100	200	400	1.000	2.000
Nu	$\nabla T = -1, \nabla C = -1$	Present Results	3.11	4.90	7.65	13.22	19.54
		Goyeau et al [0]	3.11	4.96	7.77	13.47	19.90
		Trevisan and Bejan [0]	3.27	5.61	9.69	-	-
Sh		Present Results	14.76	22.02	32.55	53.37	76.58
		Goyeau et al [0]	13.25	19.86	28.41	48.32	69.29
		Trevisan and Bejan [0]	15.61	23.23	30.76	-	-
Nu	$\nabla T = -1, \nabla C = 0$	Present Results	3.11	4.82	7.65	13.25	19.51
Nu	$\nabla T = -1, \nabla C = +1$	Present Results	3.11	4.81	7.64	13.99	19.48
Sh			14.76	22.64	32.50	53.34	76.09

Table 2: Average Nusselt and Sherwood numbers (N=0.1, Le=10, A=1).

Ra^*	Gradients Types	100	200	400	1.000	2.000
Nu	$\nabla T=-1,\nabla C=-1$	3.05	4.84	7.61	13.25	19.53
Sh		14.88	22.19	31.82	52.80	75.91
Nu	$\nabla T=-1,\nabla C=0$	3.05	4.84	7.68	13.25	19.56
Nu	$\nabla T=-1,\nabla C=+1$	3.05	4.84	7.58	13.25	19.37
Sh		14.88	22.19	32.01	52.80	75.59

Table 3: Average Nusselt and Sherwood numbers (N=1, Le=10, A=1).

Ra^*	Gradients Types	100	200	400	1.000	2.000
Nu	$\nabla T=-1,\nabla C=-1$	3.05	4.85	7.61	13.25	19.53
Sh		14.88	22.19	31.82	52.08	75.91
Nu	$\nabla T=-1,\nabla C=0$	3.05	4.84	7.65	13.25	19.56
Nu	$\nabla T=-1,\nabla C=+1$	3.05	4.85	7.58	13.92	19.53
Sh		14.88	22.19	31.82	52.08	75.91

Table 4: Average Nusselt and Sherwood numbers (N=10, Le=10, A=1).

Ra^*	Gradients Types	100	200	400	1.000	2.000
Nu	$\nabla T=-1,\nabla C=-1$	3.08	4.85	7.65	13.25	19.53
Sh		14.87	22.19	32.28	53.45	75.91
Nu	$\nabla T=-1,\nabla C=0$	3.05	4.84	7.65	13.19	19.50
Nu	$\nabla T=-1,\nabla C=+1$	3.08	4.85	7.65	13.25	19.53
Sh		14.87	22.19	32.25	53.45	75.91

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